Linear and Exponential Growth (bill payments, bank savings, population growth, retirement savings, credit card payments)

Instructor: Joanna Klukowska

CORE-UA 109

Linear growth problems from previous slides

- electricity bills

$$b(k) = p \times k + base$$

where k is the number of kWh used, p is the price per one kWh and $base$ is the base payment when the client does not use any electricity

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- car rental companies
  - Watertown

$$w(m) = 79.00$$

  - U-Hal

$$u(m) = 1.39 \times m + 29.95$$

  - Budget

$$u(m) = 0.99 \times m + 29.95$$

  - Enterprise

$$e(m) = 59.95 \text{ for } m <= 100, \text{ and}$$
$$e(m) = 0.59 \times m + 59.95 \text{ for } m > 100$$

Simple vs. Compound Interest or Linear vs. Exponential Growth

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- Which option would you select?

Which bank would you use?

year	SaveWithUs ($100.00)	BetterSavings (8%)
0	$1,000.00	$1,000.00
1	$1,100.00	$1,080.00
2	$1,200.00	$1,166.40
3	$1,300.00	$1,259.71
4	$1,400.00	$1,360.49
5	$1,500.00	$1,469.33
6	$1,600.00	$1,586.87
7	$1,700.00	$1,713.82
8	$1,800.00	$1,850.93
9	$1,900.00	$1,999.00
10	$2,000.00	$2,158.92

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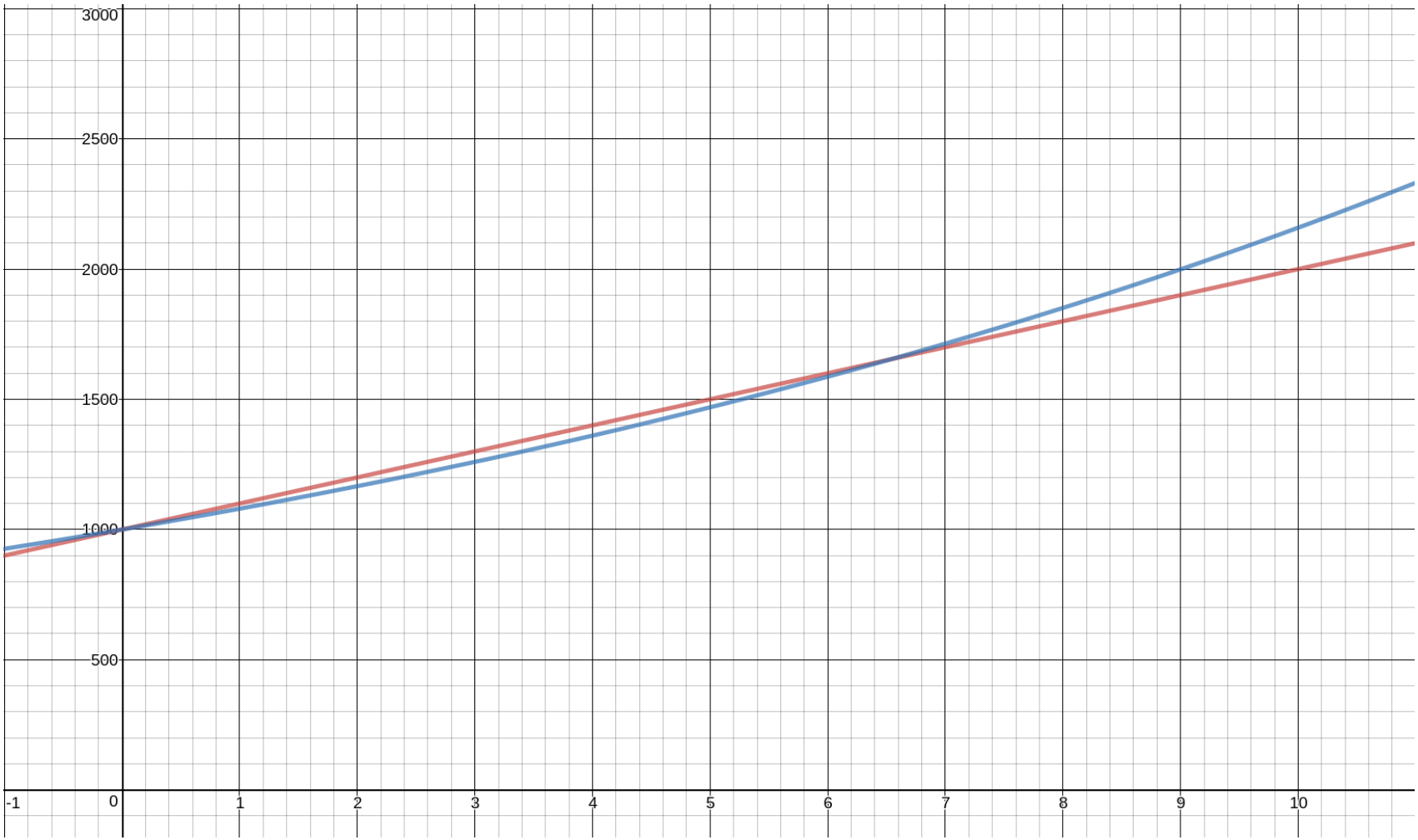
- The first model is called **simple interest** - the bank is paying a 10% interest, but it is always 10% of the original investment (so it is really a fixed amount).
- The second model is called **compound interest** - the bank is paying a 8% interest of whatever the balance of the account is (so it is changing over time).

Which bank would you use?



Graph generated and viewable at

`https://www.desmos.com/calculator/ablq5wdunm`

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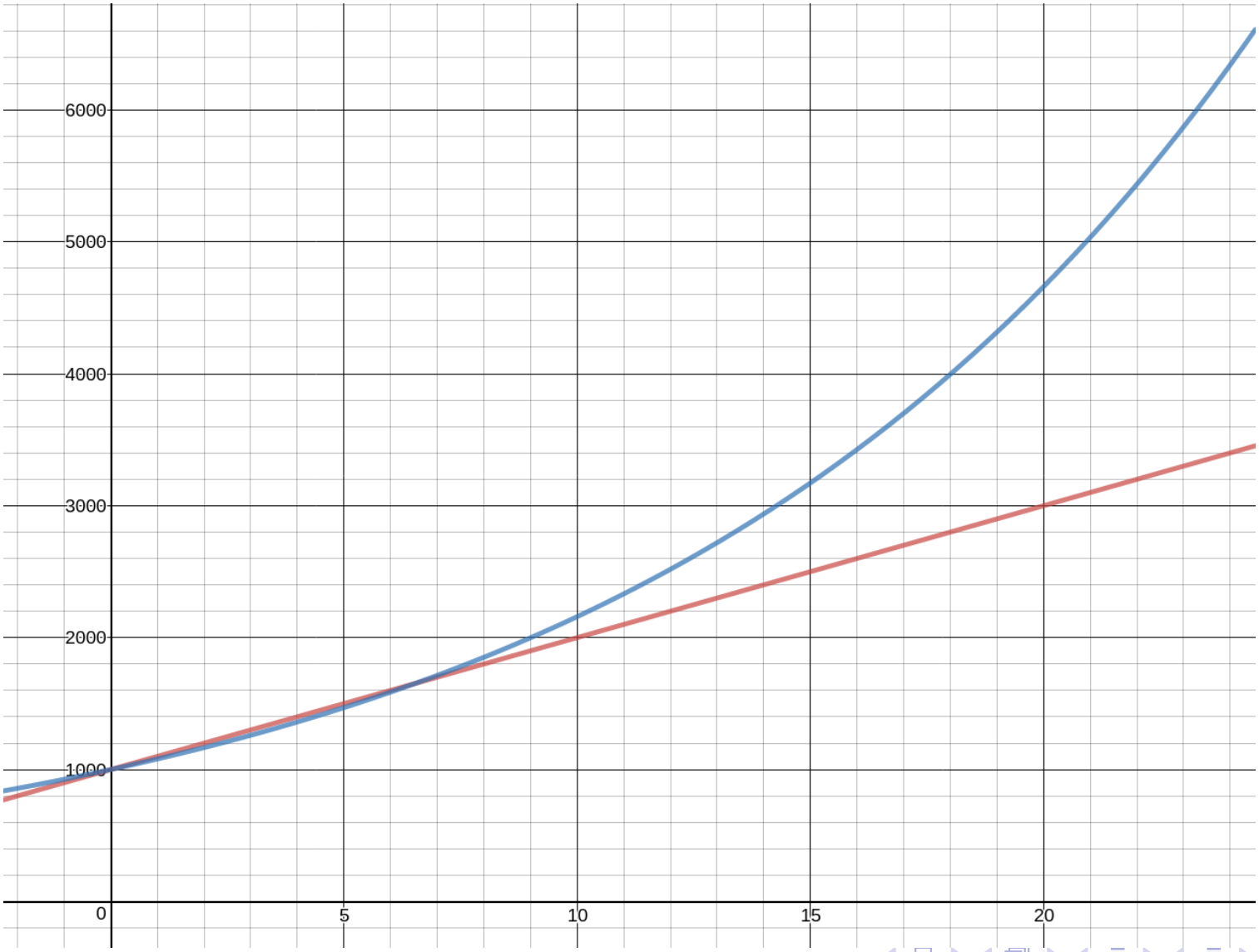

Significance of Doubling

Rice and the Chessboard Story

- The creator of the game of chess showed his invention to the ruler, the ruler was highly impressed.
- He was so impressed, in fact, that he told the inventor to name a prize of his choice.
- The inventor, being rather clever, said he would take a grain of rice on the first square of the chessboard, two grains of rice on the second square of the chessboard, four on the third square, eight on the fourth square, and so on, doubling the number of grains of rice for each successive square.
- The ruler laughed at such a modest prize, but he ordered his treasurer to count out the rice.

Rice and the Chessboard Story

- What do you think about the prize that the creator of chess asked for?

Rice and the Chessboard Story

- What do you think about the prize that the creator of chess asked for?
- Can you guess how many grains of rice will he receive?

Rice and the Chessboard Story

- ...
- The treasurer took more than a week to count the rice in the rulers store, only to notify the ruler that it would take more rice than was available in the entire kingdom.
- (Shortly thereafter, as the story goes, the inventor became the new king.)

Rice and the Chessboard Story

How many grains?

- square 1: 1 grain
- square 2: 2 grains
- square 3: 4 grains
- square 4: 8 grains
- square 5: 16 grains
- square 6: 32 grains
- square 7: 64 grains

Can you see the pattern? What would the number of grains be for a square s?

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$$\text{square } s : 2^{s-1} \text{ grains}$$

Rice and the Chessboard Story

How big is 2^{s-1} ?

- square 5: so $s = 5$ and $2^{5-1} = 16$ grains
- ...
- square 10: so $s = 10$ and $2^{10-1} = 512$ grains
- ...
- square 16: so $s = 16$ and $2^{16-1} = 32,768$ grains
- ...
- square 32: so $s = 16$ and $2^{32-1} = 2,147,483,648$ grains
- ...
- square 64: so $s = 16$ and $2^{32-1} = 9,223,372,036,854,775,808$ grains

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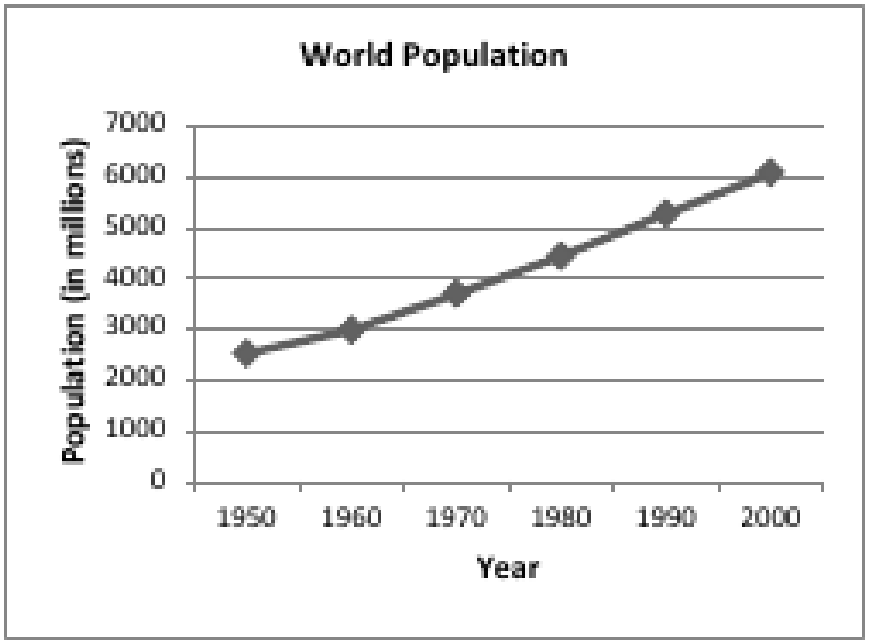
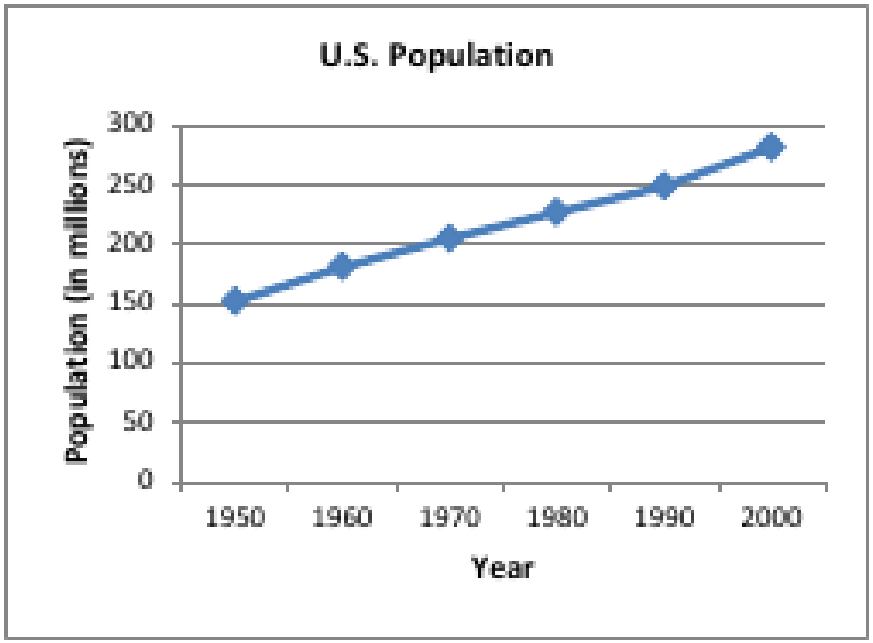
According to some source the white long grain rice yields 29,000 grains in 1 pound of rice. This gives us 318,047,311,615,682 pounds, or 159,023,655,807 tons of rice just for the last square of the chessboard.

Population Growth





Two students, Jane and Jack, are looking at the above graphs for their political science class report.

Jane: It looks like the U.S. population grew the same amount as the world population, but that cant be right, can it?

Jack: Well, I dont think they grew by the same amount, but they sure grew at about the same rate. Look at the slopes.

World vs. U.S. Population

Work with a partner to try to answer the following questions:

- Is Jane's observation correct? Why or why not?
- Is Jack's observation correct? Why or why not?
- Estimate the percent increase in world population from 1950 to 2000.
- Estimate the percent increase in U.S. population from 1950 to 2000.
- How do those two compare?
- Do the graphs above seem to indicate linear or exponential population growth? Explain your response.
- Write an explicit formula for the sequene that models the world population growth from 1950 to 2000 based on the information in the graph. Assume the population (in millions) in 1950 was 2,500 and in 2000 was 6,000. Use t to represent the number of years after 1950.
- Test the above formula by calculating the size of world population in 2000. Do you get an answer consistent with the graph? If not, you should revise the formula.
- Write a formula for U.S. population. Assume the population (in millions) in 1950 was 150 and in 2000 was 280. Use t to represent the number of years after 1950.
- Test the above formula by calculating the size of world population in 2000. Do you get an answer consistent with the graph? If not, you should revise the formula.
- Use the last formula to calculate the U.S. population in 2010. Use google to check the actual population. Are the two values consistent?

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- Assuming the same type of growth for the following generation, how many great-great-grandchildren will she have?

Human generation

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- The ancient Egypt dates back to ~3100 B.C. How many generations is it since then?
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  - Now we have ~50 centuries, so it is approximately 200 generations.

Human generations and population growth

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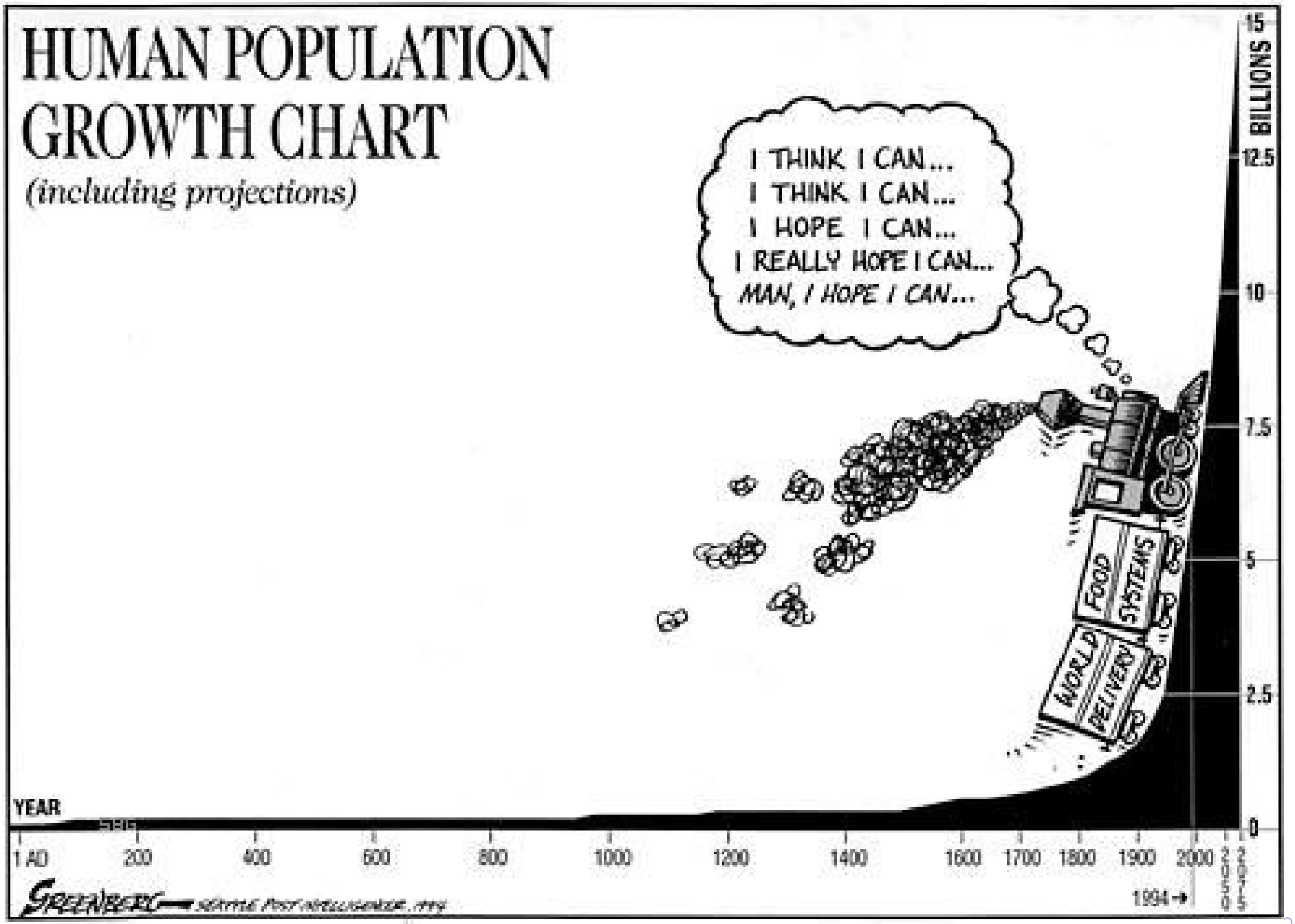
- What is the function that we can use to calculate this?
  - $r(g) = n^g$
  - g is the number of generations, n is the number of offsprings per generation, $r(g)$ is the function of number of generation that calculates the number of ”relatives”

Other Factors of Human Population

Birth rates (i.e., the number of offsprings per generation) are not the only factors that influence the current size of human population.

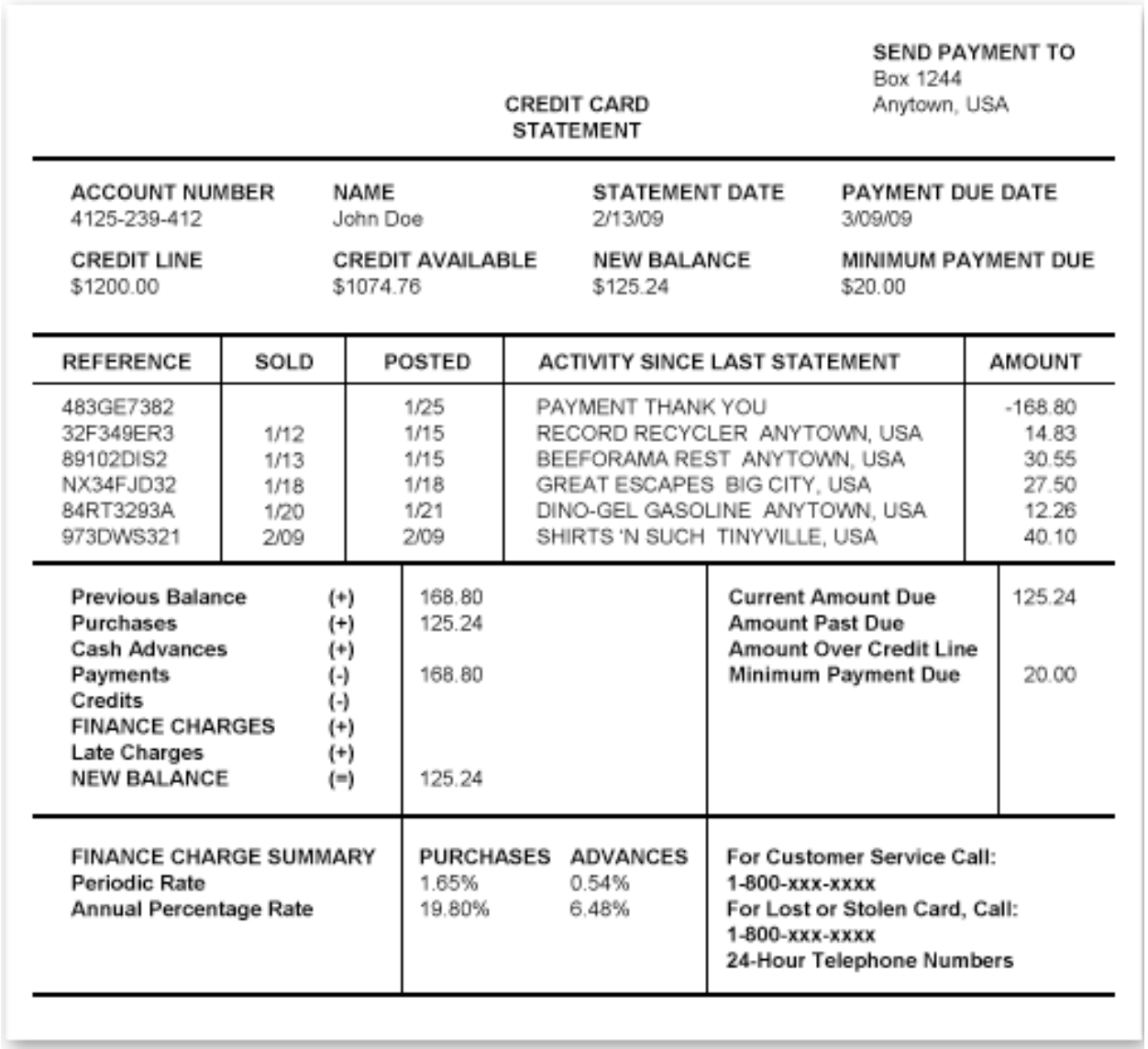
What are some other factors?


HUMAN POPULATION GROWTH CHART
(including projections)
I THINK I CAN...
I THINK I CAN...
I HOPE I CAN...
I REALLY HOPE I CAN...
MAN, I HOPE I CAN...
FOOD SYSTEMS
WORLD PEI NEED
BILLIONS
15
12.5
10
7.5
5
2.5
0
YEAR
1 AD
200
400
600
800
1000
1200
1400
1600
1700
1800
1900
2000
2050
2075
1994
GREENBERG

Borrowing and Saving

Credit Cards

- John Doe charged $125.24 to his credit card during the last statement period.
- His minimum payment due is $20.00.
- He decides to settle his debt with the credit card company by making the $20.00 monthly payments.
  He also is not going to be using this credit card anymore.
- The credit card company charges 1.65% montly interest rate on the unpaid amount.
- How many months will it take him to be debt free?
  How much extra is he going to pay the bank?



SEND PAYMENT TO
Box 1244
Anytown, USA

CREDIT CARD STATEMENT

ACCOUNT NUMBER	NAME	STATEMENT DATE	PAYMENT DUE DATE
4125-239-412	John Doe	2/13/09	3/09/09
CREDIT LINE	**CREDIT AVAILABLE**	**NEW BALANCE**	**MINIMUM PAYMENT DUE**
$1200.00	$1074.76	$125.24	$20.00

REFERENCE	SOLD	POSTED	ACTIVITY SINCE LAST STATEMENT	AMOUNT
483GE7382		1/25	PAYMENT THANK YOU	-168.80
32F349ER3	1/12	1/15	RECORD RECYCLER ANYTOWN, USA	14.83
89102DIS2	1/13	1/15	BEEFORAMA REST ANYTOWN, USA	30.55
NX34FJD32	1/18	1/18	GREAT ESCAPES BIG CITY, USA	27.50
84RT3293A	1/20	1/21	DINO-GEL GASOLINE ANYTOWN, USA	12.26
973DWS321	2/09	2/09	SHIRTS 'N SUCH TINYVILLE, USA	40.10

Previous Balance	(+)	168.80	Current Amount Due	125.24
Purchases	(+)	125.24	Amount Past Due	
Cash Advances	(+)		Amount Over Credit Line	
Payments	(-)	168.80	Minimum Payment Due	20.00
Credits	(-)			
FINANCE CHARGES	(+)			
Late Charges	(+)			
NEW BALANCE	(=)	125.24		

FINANCE CHARGE SUMMARY	PURCHASES	ADVANCES
Periodic Rate	1.65%	0.54%
Annual Percentage Rate	19.80%	6.48%

For Customer Service Call:
1-800-xxx-xxxx
For Lost or Stolen Card, Call:
1-800-xxx-xxxx
24-Hour Telephone Numbers

See

`http://www.practicalmoneyskills.com/flash/bank_tutor/index.html`

for detailed explanation of parts of the above bill.

Credit Cards

- month 0 (end of the current billing statement): statement balance is $125.24
  he pays $20.00, balance is $105.24

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  he pays \$20.00, remaining balance is \$86.98

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  he pays \$20.00, remaining balance is \$86.98
- month 2: balance with interest is \$86.98 * (1 + 0.0165) = \$88.41,
  he pays \$20.00, remaining balance is \$68.41

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- month 3: balance with interest is \$68.41 * (1 + 0.0165) = \$69.54,
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- month 3: balance with interest is \$68.41 * (1 + 0.0165) = \$69.54,
  he pays \$20.00, remaining balance is \$49.54
- month 4: balance with interest is \$49.54 * (1 + 0.0165) = \$50.36,
  he pays \$20.00, remaining balance is \$30.36

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  he pays \$20.00, remaining balance is \$86.98
- month 2: balance with interest is \$86.98 * (1 + 0.0165) = \$88.41,
  he pays \$20.00, remaining balance is \$68.41
- month 3: balance with interest is \$68.41 * (1 + 0.0165) = \$69.54,
  he pays \$20.00, remaining balance is \$49.54
- month 4: balance with interest is \$49.54 * (1 + 0.0165) = \$50.36,
  he pays \$20.00, remaining balance is \$30.36
- month 5: balance with interest is \$30.36 * (1 + 0.0165) = \$30.86,
  he pays \$20.00, remaining balance is \$10.86

Credit Cards

- month 0 (end of the current billing statement): statement balance is \$125.24
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- month 1: balance with interest is \$105.24 * (1 + 0.0165) = \$106.98,
  he pays \$20.00, remaining balance is \$86.98
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  he pays \$20.00, remaining balance is \$30.36
- month 5: balance with interest is \$30.36 * (1 + 0.0165) = \$30.86,
  he pays \$20.00, remaining balance is \$10.86
- month 6: balance with interest is \$10.86 * (1 + 0.0165) = \$11.04,
  he pays the remaining balance \$11.04

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- There is a total of \$5.81 interest paid over those six months. Does not seem like much, does it?

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  he pays the remaining balance \$11.04

- There is a total of \$5.81 interest paid over those six months. Does not seem like much, does it?
- What is the annual interest rate that this credit card charges?

Credit Cards

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Credit Cards

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  - How is the value of interest paid affected by the interest rate?

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  - What is the function that calculates the total amount that will be paid with the fixed monthly payments and fixed interest rate?

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  - How is the value of interest paid affected by the interest rate?
  - How is the value of interest paid affected by the initial balance?
  - How is the value of interest paid affected by the amount of monthly payments?

  - What is the function that calculates the total amount that will be paid with the fixed monthly payments and fixed interest rate?
  - What is the function that calculated the payment balance after each month with the fixed monthly payments and fixed interest rate?

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Simplified model:

- At the end of each year (till your retirement) you deposit $1000.00 in your retirement account.

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- How much money will there be in the account in 1 year, 2 year, 10 year, 20 years?
  - year 0: deposit $1000.00, no interest yet
  - year 1: balance plus interest $1000.00 (1 + 0.05) = $1050.00
    new deposit: $1000.00, total: $2050.00

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- How much money will there be in the account in 1 year, 2 year, 10 year, 20 years?
  - year 0: deposit \$1000.00, no interest yet
  - year 1: balance plus interest \$1000.00 (1 + 0.05) = \$1050.00
    new deposit: \$1000.00, total: \$2050.00
  - year 2: balance plus interest \$2050.00 (1 + 0.05) = \$2152.50
    new deposit: \$1000.00, total: \$3152.50

Saving For Retirement

Simplified model:

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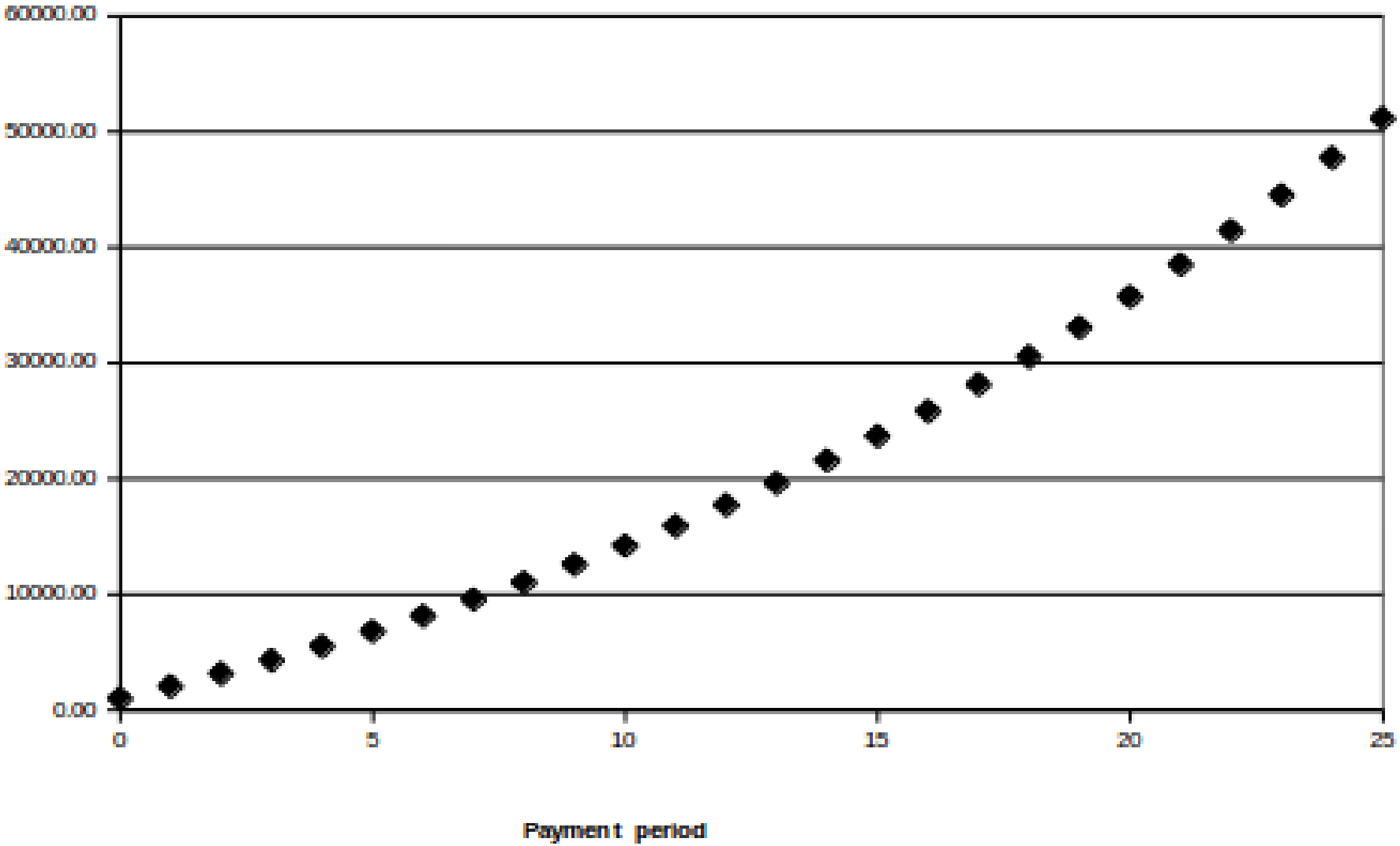
- The brokerage managing your money, guarantees that they can make 5% interest on the balance of the account each year.
- How much money will there be in the account in 1 year, 2 year, 10 year, 20 years?
  - year 0: deposit \$1000.00, no interest yet
  - year 1: balance plus interest \$1000.00 (1 + 0.05) = \$1050.00
    new deposit: \$1000.00, total: \$2050.00
  - year 2: balance plus interest \$2050.00 (1 + 0.05) = \$2152.50
    new deposit: \$1000.00, total: \$3152.50
  - ...



After 25 years of this pattern, the balance on the account is $51,113.45.

Exercises

Try it Yourself

NY State Population Growth:
The table below represents the population of the state of New York for the years 18002000. Use this information to answer the questions.

Year	Population
1800	300,000
1900	7,300,000
2000	19,000,000

- Using the year 1800 as the base year, an explicit formula for the sequence that models the population of New York is P(t) = 300 000(1.021)t , where t is the number of years after 1800. Using this formula, calculate the projected population of New York in 2010.
- Using the year 1900 as the base year, an explicit formula for the sequence that models the population of New York is P(t) = 7 300 000(1.0096)t, where t is the number of years after 1900. Using this formula, calculate the projected population of New York in 2010.
- Using the Internet (or some other source), find the population of the state of New York according to the 2010 census. Which formula yielded a more accurate prediction of the 2010 population?
- (Extra Challenge) Figure out how the formulas in the above questions were derived.

Try it Yourself

- A rare coin appreciates at a rate of 5.2% a year. If the initial value of the coin is $500, after how many years will its value cross the $3,000 mark? Show the formula that models the value of the coin after t years.
- A local college has increased its number of graduates by a factor of 1.045 over the previous year for every year since 1999. In 1999, 924 students graduated. What explicit formula models this situation? Approximately how many students will graduate in 2014?
- A three-bedroom house in Burbville sold for $190,000. If housing prices are expected to increase 1.8% annually in that town, write an explicit formula that models the price of the house in t years. Find the price of the house in 5 years.

Try It Yourself

Retirement savings

- What is the function that would calculate the value of your retirement assuming a fixed interest rate and fixed yearly deposit?
- How does this formula compare to the credit card payment formula?
- How would the value of your retirement account change if the payments were made monthly and interest compounded monthly?
- How much would be in your retirement account if you were earning interest equal to the interest rate charged by the banks on your credit card balance?